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Supersymmetric Wilson Loops in IIB Matrix Model

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Abstract

We show that the supersymmetric Wilson loops in IIB matrix model give a transition operator from reduced supersymmetric Yang-Mills theory to supersymmetric space-time theory. In comparison with Green-Schwarz superstring we identify the supersymmetric Wilson loops with the asymptotic states of IIB superstring. It is pointed out that the supersymmetry transformation law of the Wilson loops is the inverse of that for the vertex operators of massless modes in the $U(N)$ open superstring with Dirichlet boundary condition.

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1 Introduction

For a long time it has been hoped that the large N gauge theory [1] will give a nonperturbative definition of string theory. In the begining of '90 2D string has solved exactly in terms of matrix models [2] and many works have been carried out in this field [3]. The identification with the continuum theory was done in the direct calculations of amplitudes [4] and then was completed using the W_∞ symmetry [5].

Recently more realistic matrix models called M(atrix) theory [6] and IIB matrix theory [7] (and also see [8 – 15]) have been proposed, which are described in terms of the D-particles and the D-instantons [16, 17]. In this paper we study the IIB matrix model, which is hoped to give the type IIB superstring. In this case, other than 2D string, the oscillation modes with continuous momenta will arise. The aim of this paper is to clarify how the oscillation modes arise in the IIB matrix theory. We study such an issue using the supersymmetry.

The Wilson loops will describe the operators which create and annihilate strings [7, 15]. We here introduce supersymmetric Wilson loops in IIB matrix theory and identify it with the asymptotic state of superstring. To carry out the program we first study the supersymmetry transformation law of the wave function of IIB superstring which is constructed by acting the vertex operator of the D-instanton [18] on the boundary state [17]. We then show that the supersymmetric Wilson loop just has the same property as that the state of superstring has, where the supersymmetry transformation of reduced super Yang-Mills theory acts on it as a counterpart of that of world-sheet theory.

2 The state of IIB superstring

We first construct the eigenstate of Hamiltonian using Green-Schwarz superstring quantized in the light-cone gauge [19, 20] and then discuss its supersymmetry transformation law.

Let us consider cylinder frame with Dirichlet boundary at $\tau = 0$. The boundary state is defined by the conditions

$$\partial_\sigma X^\mu(\sigma)|B>= 0 \ , \qquad S^{+a}(\sigma)|B>= 0 \ , \tag{2.1}$$

where $S^{\pm a} = \frac{1}{\sqrt{2}}(S^a \pm i\tilde{S}^a)$ and $\mu = (+,-,I) \quad I = 1,\cdots,8$. This is the D-instanton state discussed in [17, 18]. In the following we mainly use the notations and conventions of ref. [18]. The conditions can be solved easily and we obtain

$$|B>= \exp\Big[\sum_{n=1}^{\infty}\Big(\frac{1}{n}\alpha^I_{-n}\tilde{\alpha}^I_{-n} - iS^a_{-n}\tilde{S}^a_{-n}\Big)\Big]|B_0> \ , \tag{2.2}$$

where $|B_0> = |I>|I> -i|\dot{a}>|\dot{a}>$. The mode expansions of string coordinates are defined by

$$\begin{aligned} X^I(\tau,\sigma) &= x^I + p^I\tau + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\Big(\alpha^I_n \mathrm{e}^{-2in\pi(\tau-\sigma)} + \tilde{\alpha}^I_n \mathrm{e}^{-2in\pi(\tau+\sigma)}\Big) \ , \\ S^a(\tau,\sigma) &= \sum_n S^a_n \mathrm{e}^{-2in\pi(\tau-\sigma)} \ , \\ \tilde{S}^a(\tau,\sigma) &= \sum_n \tilde{S}^a_n \mathrm{e}^{-2in\pi(\tau+\sigma)} \ . \end{aligned} \tag{2.3}$$

The vertex operator of (single) D-instanton [18] is defined in terms of the broken currents $\partial_\tau X^\mu$ and S^- for translational invariance and supersymmtery in the form

$$V(x^\mu(\sigma),\eta(\sigma)) = \int_0^\pi \frac{d\sigma}{\pi}\Big\{x_\mu(\sigma) - i\bar{\eta}(\sigma)\Gamma_\mu\theta\Big\}\partial_\tau X^\mu \ , \tag{2.4}$$

where $\theta = (2^{\frac{1}{4}}2i\sqrt{p^+})^{-1}\Gamma^+S^-$. We here consider the σ-dependent functions $x^\mu(\sigma)$ and $\eta(\sigma)$. In the light-cone gauge defined by $x^+(\sigma) = x^+ = \tau$ and $\Gamma^+\eta = 0$, the vertex operator reduces to the simple form

$$V(x^\mu,\eta) = \int_0^\pi \frac{d\sigma}{\pi}\Big\{x^I(\sigma)\partial_\tau X^I - x^-(\sigma)p^+ + i2^{-\frac{1}{4}}\sqrt{p^+}\eta^a(\sigma)S^{-a}\Big\} \ . \tag{2.5}$$

Let us consider the Wilson loop operator $w = \exp(-iV)$ and act it on the boundary state. Using the Baker-Campbell-Hausdorff formula we can obtain the following state:

$$\begin{aligned} &|x,\eta>= w|B> \\ &= \exp\Big(-ix^I_0 p^I + ix^-_0 p^+ + 2^{-\frac{1}{4}}\sqrt{p^+}\eta^a_0 S^{-a}_0\Big) \\ &\times \exp\Big[\sum_{n=1}^{\infty}\Big\{-nx^I_n x^I_{-n} - 2i\Big(x^I_n\alpha^I_{-n} + x^I_{-n}\tilde{\alpha}^I_{-n}\Big) + \frac{1}{n}\alpha^I_{-n}\tilde{\alpha}^I_{-n}\Big\}\Big] \\ &\times \exp\Big[\sum_{n=1}^{\infty}\Big\{\frac{p^+}{2\sqrt{2}}\eta^a_n\eta^a_{-n} + 2^{\frac{1}{4}}\sqrt{p^+}\Big(\eta^a_n S^a_{-n} - i\eta^a_{-n}\tilde{S}^a_{-n}\Big) - iS^a_{-n}\tilde{S}^a_{-n}\Big\}\Big]|B_0> \ , \end{aligned} \tag{2.6}$$

where the mode expansions of x and η are defined by

$$x^I(\sigma) = \sum_n x_n^I \mathrm{e}^{2in\sigma} \ , \qquad \eta^a(\sigma) = \sum_n \eta_n^a \mathrm{e}^{2in\sigma} \ . \tag{2.7}$$

This state satisfies the boundary conditions $X^I(\sigma)|x,\eta>= x^I(\sigma)|x,\eta>$ and $S^{+a}(\sigma)|x,\eta>= -2^{-\frac{1}{4}}\sqrt{p^+}\eta^a(\sigma)|x,\eta>$. From the expression of vertex operator, the operators $\partial_\tau X^I(\sigma)$ and $S^{-a}(\sigma)$ are described in terms of the functional derivatives of $x^I(\sigma)$ and $\eta^a(\sigma)$, respectively.

The light-cone Hamiltonian is given by

$$H = \frac{1}{2p^+}\int_0^\pi \frac{d\sigma}{\pi}\Big[(\partial_\tau X^I)^2 + (\partial_\sigma X^I)^2 - iS^{+a}\partial_\sigma S^{+a} - iS^{-a}\partial_\sigma S^{-a}\Big] \ . \tag{2.8}$$

Therefore the state $|x,\eta,\tau>= \mathrm{e}^{i\tau H}|x,\eta>$ satisfies the Schrödinger equation

$$-i\frac{\partial}{\partial\tau}|x,\eta,\tau>= H|x,\eta,\tau>= h|x,\eta,\tau> \tag{2.9}$$

where

$$\begin{aligned} h = \frac{1}{2p^+}\int_0^\pi \frac{d\sigma}{\pi}\Big[&-\pi^2\Big(\frac{\delta}{\delta x^I(\sigma)}\Big)^2 + (\partial_\sigma x^I)^2 \\ &-i\frac{\sqrt{2}\pi^2}{p^+}\frac{\delta}{\delta\eta^a(\sigma)}\partial_\sigma\frac{\delta}{\delta\eta^a(\sigma)} - i\frac{p^+}{\sqrt{2}}\eta^a(\sigma)\partial_\sigma\eta^a(\sigma)\Big] \end{aligned} \tag{2.10}$$

and the wave function is defined by $\Phi^\star(x,\eta,\tau) =< \Phi|x,\eta,\tau>$.

In the last of this section we discuss supersymmetry transformation law of the state. The supersymmetry transformation of vertex operator with respect to the unbroken supercharge Q^+ is translated into the supersymmetry transformation on x^μ and η. Using the equations

$$\begin{aligned} \hat{\delta}_\alpha^{(+)}(\partial_\tau X^I) &= 2^{\frac{1}{4}}\frac{i}{\sqrt{p^+}}\alpha^{\dot{a}}\gamma^I_{a\dot{a}}\partial_\sigma S^{-a} \ , \\ \hat{\delta}_\alpha^{(+)}S^{-a} &= 2^{\frac{1}{4}}\sqrt{2p^+}\alpha^a + 2^{\frac{1}{4}}\frac{1}{\sqrt{p^+}}\alpha^{\dot{a}}\gamma^I_{a\dot{a}}\partial_\tau X^I \ , \end{aligned} \tag{2.11}$$

where $\hat{\delta}_\alpha^{(+)} = [\alpha^a Q^{+a} + \alpha^{\dot{a}}Q^{+\dot{a}}, \quad]$, we obtain the equation

$$\hat{\delta}_\alpha^{(+)}V(x^\mu,\ \eta) = V(\bar{\delta}_\alpha x^\mu,\ \bar{\delta}_\alpha\eta) \ , \tag{2.12}$$

where

$$\begin{aligned}
&\bar{\delta}_\alpha x^I(\sigma) = -i\alpha^{\dot{a}}\gamma^I_{a\dot{a}}\eta^a(\sigma) \ , \\
&\bar{\delta}_\alpha x_0^- = i\sqrt{2}\alpha^a\eta_0^a \ , \\
&\bar{\delta}_\alpha \eta^a(\sigma) = -\frac{\sqrt{2}}{p^+}\alpha^{\dot{a}}\gamma^I_{a\dot{a}}\partial_\sigma x^I(\sigma) \ .
\end{aligned} \tag{2.13}$$

Using this we obtain $\hat{\delta}^{(+)}w = \bar{\delta}w$. Thus the Wilson loop gives the transition operator from the world-sheet theory to the space-time theory.

3 Supersymmetric Wilson loops in IIB matrix model

The world-sheet is regulated in the large N picture. The reduced supersymmetric Yang-Mills theory will play an fundamental role to make a world-sheet. The identification of gauge fields with space-time coordinates will gives a non-pertubative definition of the type IIB superstring [7]. The states in IIB matrix model should have the same supersymmetry transformation law as that the continuum theory has. In this section we show that the supersymmetric Wilson loop just has the expected property. So we identify it with the IIB superstring state (in momentum space).

The supersymmetric Wilson loop operator we introduce here is

$$w(C) = tr\prod_{j=1}^{M} U_j \ , \qquad U_j = \mathrm{e}^{-i\epsilon V_j} \ , \tag{3.1}$$

where V_j is defined using the superfield in the form $V_j = k_j^\mu \mathcal{A}_\mu(\lambda_j)$, where $\mathcal{A}_\mu(\lambda_j) = \mathrm{e}^{\bar{\lambda}_j G} A_\mu \mathrm{e}^{-\bar{\lambda}_j G}$. G is the generator of the supersymmetric Yang-Mills transformation

$$\delta_\alpha A_\mu = i\bar{\alpha}\Gamma_\mu\Psi \ , \qquad \delta_\alpha\Psi = -\frac{i}{2}[A_\mu, A_\nu]\Gamma^{\mu\nu}\alpha \ . \tag{3.2}$$

Thus we define

$$\begin{aligned}
V_j = k_j^\mu \Big(& A_\mu - i\bar{\Psi}\Gamma_\mu\lambda_j + \frac{1}{4}[A_\nu, A_\lambda]\bar{\lambda}_j\Gamma_\mu\Gamma^{\nu\lambda}\lambda_j \\
& -\frac{i}{6}[A_\nu, \bar{\Psi}]\Gamma_\lambda\lambda_j\bar{\lambda}_j\Gamma_\mu\Gamma^{\nu\lambda}\lambda_j + \cdots \Big) \ .
\end{aligned} \tag{3.3}$$

In the following we work in the light-cone gauge [2]

$$k_j^+ = k^+ \ , \qquad \Gamma^+ \lambda_j = 0 \ . \tag{3.4}$$

Then we can see that the following supersymmetry transformation is realized:

$$\delta_\alpha w(C) = \bar{\delta}_\alpha w(C) \ , \tag{3.5}$$

where

$$\begin{aligned} \bar{\delta}_\alpha k_j^I &= 2i\alpha^{\dot{a}} \gamma^I_{a\dot{a}} \Delta\lambda_j^a \ , \\ \bar{\delta}_\alpha k^+ &= 0 \ , \\ \bar{\delta}_\alpha \lambda_j^a &= \alpha^a + \frac{1}{\sqrt{2}k^+} k_j^I \gamma^I_{a\dot{a}} \alpha^{\dot{a}} \end{aligned} \tag{3.6}$$

and $\Delta\lambda_j^a = \frac{1}{\epsilon}(\lambda_{j+1}^a - \lambda_j^a)$. The transformation of k_j^- is defined through the equation

$$k_j^- = \frac{1}{2k^+}(k_j^I)^2 - i\sqrt{2}\lambda_j^a \Delta\lambda_j^a \tag{3.7}$$

in the form $\bar{\delta}_\alpha k_j^- = -i\sqrt{2}\alpha^a \Delta\lambda_j^a - i\Delta(\frac{1}{k^+} k_j^I \lambda_j^a \gamma^I_{a\dot{a}} \alpha^{\dot{a}})$. In the continuum limit $k_j^\mu \to k^\mu(\sigma)$ and $\Delta\lambda_j^a \to \partial_\sigma \lambda^a(\sigma)$, this just corresponds to the supersymmetry transformation in momentum space derived in the continuum theory (2.13) [3]. In this case the supersymmetric Yang-Mills transformation δ just corresponds to the variation of world-sheet theory $\hat{\delta}^{(+)}$. The constraint (3.7) corresponds to the boundary condition $\partial_\tau X^- |B\rangle = \frac{1}{2p^+}[(\partial_\tau X^I)^2 - iS^{-a}\partial_\sigma S^{-a}]|B\rangle$.

The above transformation law can be proved in the following. In the light-cone gauge the matrix V_j is described in $SO(8)$ notation as

$$V_j = V_j^0 + V_j^1 + V_j^2 + V_j^3 + \cdots \ , \tag{3.8}$$

where

$$V_j^0 = k_j^I A^I - k^+ A^- - k_j^- A^+ \ ,$$

[2] The covariant description of supersymmetry does not go well, where the constraint equation which serves as eq. (3.7) is not known.

[3] The transition function between coordinate space to momentum space is given by $w_f = \exp\{i(-x^+ k_0^- - x_0^- k^+ + \int \frac{d\sigma}{\pi} x^I(\sigma) k^I(\sigma) + i\sqrt{2}k^+ \int \frac{d\sigma}{\pi} \eta^a(\sigma)\lambda^a(\sigma))\}$ such that $\bar{\delta}^{(c)} w_f = \bar{\delta}^{(m)} w_f$, where $\bar{\delta}^{(c)}$ and $\bar{\delta}^{(m)}$ are defined by (2.13) and (3.6), respectively.

$$
\begin{aligned}
V_j^1 &= -i\sqrt{2}k^+\lambda_j^a\psi^a - i\sqrt{2}k_j^I\lambda_j^a\gamma^I_{a\dot{a}}\psi^{\dot{a}} \ , \\
V_j^2 &= \frac{k^+}{2\sqrt{2}}[A^I, A^J]\lambda_j^a\gamma^{IJ}_{ab}\lambda_j^b - \frac{1}{2\sqrt{2}}k_j^I[A^+, A^J]\lambda_j^a(\gamma^I\gamma^J)_{ab}\lambda_j^b \ , \\
V_j^3 &= -\frac{i}{3\sqrt{2}}k^+[A^I, \psi^{\dot{a}}\gamma^J_{a\dot{a}}\lambda_j^a]\lambda_j^b\gamma^{IJ}_{bc}\lambda_j^c + \frac{i}{3\sqrt{2}}k_j^I[A^+, \psi^{\dot{a}}\gamma^J_{a\dot{a}}\lambda_j^a]\lambda_j^b(\gamma^I\gamma^J)_{bc}\lambda_j^c \ .
\end{aligned}
\tag{3.9}
$$

The supersymmetry transformation is described in the $SO(8)$ notation as

$$
\begin{aligned}
\delta_\alpha A^I &= -i\alpha^{\dot{a}}\gamma^I_{a\dot{a}}\psi^a - i\alpha^a\gamma^I_{a\dot{a}}\psi^{\dot{a}} \ , \\
\delta_\alpha A^+ &= i\sqrt{2}\alpha^{\dot{a}}\psi^{\dot{a}} \ , \\
\delta_\alpha A^- &= i\sqrt{2}\alpha^a\psi^a \ , \\
\delta_\alpha \psi^a &= \frac{i}{2}[A^I, A^J]\gamma^{IJ}_{ab}\alpha^b - i\sqrt{2}[A^-, A^I]\gamma^I_{a\dot{a}}\alpha^{\dot{a}} + i[A^+, A^-]\alpha^a \ , \\
\delta_\alpha \psi^{\dot{a}} &= \frac{i}{2}[A^I, A^J]\gamma^{IJ}_{\dot{a}\dot{b}}\alpha^{\dot{b}} - i\sqrt{2}[A^+, A^I]\gamma^I_{a\dot{a}}\alpha^a - i[A^+, A^-]\alpha^{\dot{a}} \ .
\end{aligned}
\tag{3.10}
$$

Let us first consider the variation of V_j^0 under the supersymmetry transformation δ. We can easily obtain the following equation:

$$
\delta_\alpha V_j^0 = \bar{\delta}_\alpha(V_j^0 + V_j^1) + \Delta f_j^0 \ , \tag{3.11}
$$

where

$$
f_j^0 = -2i\alpha^{\dot{a}}\gamma^I_{a\dot{a}}\lambda_j^a A^I - i\sqrt{2}\alpha^a\lambda_j^a A^+ + \frac{i}{k^+}k_j^I\alpha^{\dot{a}}\gamma^I_{a\dot{a}}\lambda_j^a A^+ \ . \tag{3.12}
$$

In the next step we obtain

$$
\delta_\alpha(V_j^0 + V_j^1) = \bar{\delta}_\alpha(V_j^0 + V_j^1 + V_j^2) + Y_j^0 + \Delta(f_j^0 + f_j^1) \ , \tag{3.13}
$$

where

$$
Y_j^0 = i[f_j^0, V_j^0] \tag{3.14}
$$

and

$$
f_j^1 = i\frac{\sqrt{2}}{3}[A^+, A^J]\alpha^{\dot{a}}\gamma^I_{a\dot{a}}\lambda_j^a\lambda_j^b(\gamma^I\gamma^J)_{bc}\lambda_j^c \ . \tag{3.15}
$$

In general we will obtain the equation

$$
\delta_\alpha V_j = \bar{\delta}_\alpha V_j + Y_j + \Delta f_j \ , \tag{3.16}
$$

where $Y_j = Y_j^0 + Y_j^1 + \cdots$ and $f_j = f_j^0 + f_j^1 + \cdots$.

We will see that the extra term Y_j is canceled in the Wilson loop. Let us consider the supersymmetry transformation of the Wilson loop. Using the relation (3.16) we obtain

$$\begin{aligned} \delta_\alpha w(C) &= -tr \sum_{l=1}^{M} \Big(\prod_{j=1}^{l-1} U_j \Big) i\epsilon \delta_\alpha V_l \Big(\prod_{j=l}^{M} U_j \Big) \\ &= -tr \sum_{l=1}^{M} \Big(\prod_{j=1}^{l-1} U_j \Big) i\epsilon \Big(\bar{\delta}_\alpha V_l + Y_l + \Delta f_l \Big) \Big(\prod_{j=l}^{M} U_j \Big) . \end{aligned} \tag{3.17}$$

Noting that $\Delta f_l = \frac{1}{\epsilon}(f_{l+1} - f_l)$ and f_l is the matrix such that $\frac{1}{\epsilon} f_l U_{l-1} = U_{l-1}(\frac{1}{\epsilon} f_l - i[f_l, V_{l-1}] + o(\epsilon))$, we get the following expression:

$$\delta_\alpha w(C) = -tr \sum_{l=1}^{M} \Big(\prod_{j=1}^{l-1} U_j \Big) i\epsilon \Big(\bar{\delta}_\alpha V_l + Y_l - i[f_l, V_{l-1}] \Big) \Big(\prod_{j=l}^{M} U_j \Big) . \tag{3.18}$$

As shown in the above calculation (3.14), Y_l cancels $i[f_l, V_{l-1}]$ iteratively in the continuum limit. This cancellation is an analogy of that by contact terms in open superstring with Chan-Paton factor [18, 21]. Thus we can prove the supersymmetry transformation law of the Wilson loop.

This is the inverse picture of supersymmetry transformation law of the vertex operator for massless mode in the $U(N)$ Dirichlet open superstring derived in [18], where the roles of the world-sheet theory and space-time theory are exchanged. Supersymmetric Yang-Mills theory now plays an role of world-sheet theory, not of the space-time one.

4 The S-matrix

In the previous section we discussed the supersymmetric Wilson loop in IIB matrix theory. We proposed that, in the symmetrical point of view, it corresponds to the asymptotic state of IIB superstring. The correlation function of the Wilson loops is defined by

$$< w(k_1, \lambda_1) \cdots w(k_L, \lambda_L) >= \int dAd\Psi w(C_1) \cdots w(C_L) \exp(-S) \tag{4.1}$$

where S is the reduced supersymmetric Yang-Mills acton. The continuum limit is defined by $M\epsilon = 1$ and $g^2 N = 1$, where g is the gauge coupling

behaived as $g \sim \epsilon$. The momentum conservation comes from the integration over the $U(1)$ part in $U(N)$ matrix. The $U(1)$ part of A^- integral gives the delta function $\delta(k_1^+ + \cdots + k_L^+)$ and others gives $\delta(\epsilon \sum_{j=1}^M k_{1j}^\mu + \cdots + \epsilon \sum_{j=1}^M k_{Lj}^\mu)$, where $\mu = -, I$. In the continuum limit these give the momentum conservations of zero-modes, $k_0^\mu = \int \frac{d\sigma}{\pi} k^\mu(\sigma)$.

Thus the S-matrix is defined by attaching the wave functions of oscillation modes $\Phi(k, \lambda)$ in the form:

$$S_{i \to f} = \int \prod_{q=1}^{L} \frac{1}{\sqrt{|k_q^+|}} [D' k_q^I][D\lambda_q^a] \Phi(k_q, \lambda_q) < w(k_1, \lambda_1) \cdots w(k_L, \lambda_L) > \quad (4.2)$$

where $[D'k^I][D\lambda^a]$ means the integration over transverse oscillation modes and the prime stands for the exclusion of the zero-modes. Incoming states (outgoing states) are defined by the Wilson loops with k^+ positive (negative).

Finally we briefly comment on the Schwinger-Dyson equation of the followin type:

$$\int dA d\Psi \frac{\partial}{\partial A^{+\alpha}} \sum_{l=1}^{M} tr \Big(\prod_{j=1}^{l-1} U_j \ t^\alpha \ \prod_{j=l}^{M} U_j \Big) \exp(-S) = 0 \ . \quad (4.3)$$

This is likely to correspond to the equation $< 0|H|\Phi >= 0$ in the continuum theory. Here the effects of the terms corresponding to $(\partial_\sigma x^I)^2$ and $\eta^a \partial_\sigma \eta^a$ in the Hamiltonian will be included in the derivative of action with respect to A^+. This is similar to the picture of the Hartle-Hawking wave function.

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References

[1] G. 't Hooft, Nucl. Phys. **B72** (1974) 461.

[2] E. Brezin and V. Kazakov, Phys. Lett. **B236** (1990) 144; M. Douglas and S. Shenker, Nucl. Phys. **B335** (1990) 635; D. Gross and A. Migdal, Phys. Rev. Lett. **64** (1990) 127.

[3] E. Brezin and S. R. Wadia, *The large N expansion in quantum field theory and statistical physics: from spin systems to two-dimensional gravity* World Scientific (1993).

[4] M. Goulian and M. Li, Phys. Rev. Lett. **66** (1991) 2051; Y. Kitazawa, Phys. Lett. **B265** (1991) 262; P. DiFranceso and D. Kutasov, Phys. Lett. **B261** (1991) 385; V. Dotsenko, Mod. Phys. Lett. **A6** (1991) 3601; K. Aoki and E. D'Hoker, Mod. Phys. Lett. **A7** (1992) 235.

[5] K. Hamada, Phys. Lett. **B324** (1994) 141; **B462** (1996) 192.

[6] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, Phys. Rev. **D55** (1997) 5112, hep-th/9610043.

[7] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, *A Large-N Reduced Model as Superstring*, hep-th/9612115.

[8] V. Periwal, Phys. Rev. **D55** (1997) 1711, hep-th/9611103.

[9] M. Li, *Strings from IIB Matrices*, hep-th/961222.

[10] A. Tseytlin, *On Non-abelian Generalization of Born-Infeld Action in String Theory*, hep-th/9701125; I. Chepelev and A. Tseytlin, *Interactions of Type IIB D-branes from D-instanton Matrix Model*, hep-th/9705120.

[11] I. Chepelev, Y. Makeenko and K. Zarembo, *Properties of D-branes in Matrix Model of IIB Superstring*, hep-th/9701151.

[12] A. Fayyazuddin and D. J. Smith, *P-Brane Solutions in IKKT IIB Matrix Theory*, hep-th/9701168.

[13] A. Fayyazuddin, Y. Makeenko, P. Olesen, D. J. Smith and K. Zarembo, *Towards a Non-perturbative Formulation of IIB Superstrings by Matrix Models*, hep-th/9703038.

[14] T. Yoneya, *Schild Action and Space-Time Uncertainty Principle in String Theory*, hep-th/9703078.

[15] M. Fukuma, H. Kawai, Y. Kitazawa and A. Tsuchiya, *String Field Theory from IIB Matrix Model*, hep-th/9705128.

[16] J. Polchinski, *TASI Lectures on D-branes*, hep-th/9611050.

[17] M. B. Green and M. Gutperle, Nucl. Phys. **B476** (1996) 484, hep-th/9604091.

[18] K. Hamada, *Vertex Operators for Super Yang-Mills and Multi D-branes in Green-Schwarz Superstring*, hep-th/9612234, to appear in Nucl. Phys. B.

[19] M. B. Green, J. H. Schwarz and L. Brink, Nucl. Phys. **B219** (1983) 437.

[20] M. B. Green, J. H. Schwarz and E. Witten, *Superstring theory* (CUP, 1987).

[21] M. B. Green and N. Seiberg, Nucl. Phys. **B299** (1988) 559.